3. Equation of a line in plane

$(x_1 y_1)$, $(x_2 y_2)$

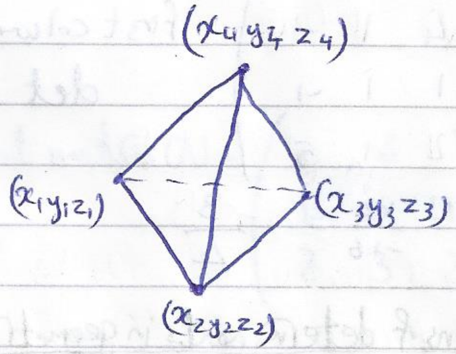
$$\det = \begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = \text{equation of straight line}$$

$y = mx + c$

or $ax + by = c$

4. Volume of tetrahedron

$$\text{Volume} = \pm \frac{1}{6} \det \begin{pmatrix} x_1 & y_1 & z_1 & 1 \\ x_2 & y_2 & z_2 & 1 \\ x_3 & y_3 & z_3 & 1 \\ x_4 & y_4 & z_4 & 1 \end{pmatrix}$$



4. Vector Spaces

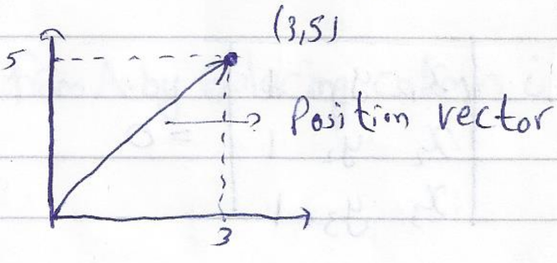
Def. of sets: A set is a collection of things that have common properties and are called elements of set.

e.g. $S = \{x : 1 \leq x \leq 6,\ x \text{ is an integer}\}$

in plane $\mathbb{R}^2$

Any ordered pair of numbers represent a vector in the plane

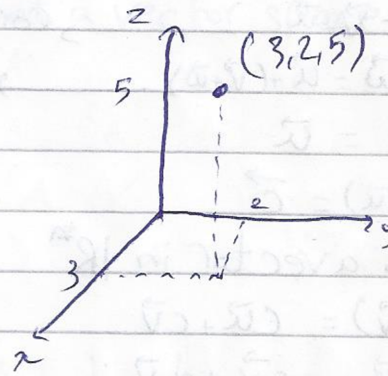
$V = (x, y)$ e.g. $v = (3, 5)$



In space $\mathbb{R}^3$

Set of all ordered triples of real numbers $V=(x, y, z)$

eg. $V=(3,2,5)$

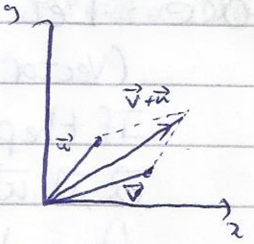


In plane

1. Vector addition

$\vec{V}=(V_1, V_2) \qquad \vec{u}=(u_1, u_2)$

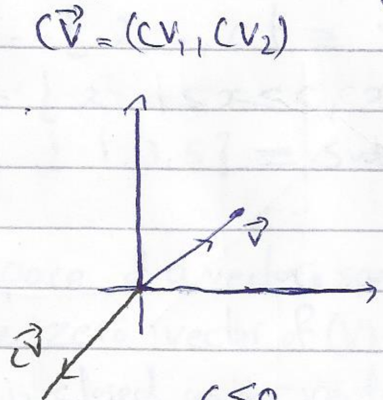
$\vec{V}+\vec{u}=(V_1+u_1, V_2+u_2)$ or using graph



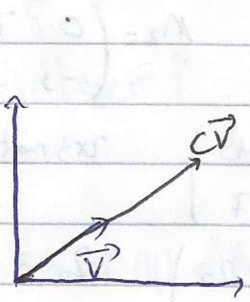
2. Scalar multiplication

$\vec{V}=(V_1, V_2) \qquad C=$ any scalar

$C\vec{V}=(CV_1, CV_2)$



$C<0$



$C>0$

properties of vector addition and scalar multiplication in $\mathbb{R}^n$

Let $\vec{u}, \vec{v}, \vec{w}$ be vectors in $\mathbb{R}^n$ and let c, d be two scalars

1. $\vec{u} + \vec{v}$ is a vector in $\mathbb{R}^n$ $\longrightarrow$ closed under addition
2. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$ $\longrightarrow$ commutative property under addition
3. $(\vec{u}+\vec{v}) + \vec{w} = \vec{u} + (\vec{v}+\vec{w})$ $\longrightarrow$ associative under addition
4. $\vec{u} + \vec{0} = \vec{u}$
5. $\vec{u} + (-\vec{u}) = \vec{0}$
6. $c\vec{u}$ is a vector in $\mathbb{R}^n$
7. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$
8. $(c+d)\vec{v} = c\vec{v} + d\vec{v}$
9. $c(d\vec{v}) = cd(\vec{v}) = d(c\vec{v})$
10. $1\vec{u} = \vec{u}$

Def of vectorspace : Let V be a set on which two operation (vector addition and scalar multiplication) are defined. If the previous listed properties are satisfied for any $\vec{u}, \vec{v}, \vec{w}$ vectors in V and every scalar c and d then V is called a vector space

Ex. The set of all 2x3 matrices

$$M_1 = \begin{pmatrix} 2 & 3 & 5 \\ 8 & 4 & 6 \end{pmatrix} \qquad M_2 = \begin{pmatrix} 0 & 7 & 8 \\ -3 & -2 & 1 \end{pmatrix}$$

① $M_1 + M_2 = \begin{pmatrix} 2 & 10 & 13 \\ 5 & 2 & 7 \end{pmatrix}$ 2x3 matrix

② $5M_1 = \begin{pmatrix} 10 & 15 & 25 \\ 40 & 20 & 30 \end{pmatrix}$ 2x3 matrix

③ $M_1 + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} = M_1$

The properties are satisfied $\therefore$ the set of all 2x3 matrices ~~are~~ is a vector space

Ex. The set of integers
$\frac{1}{2}(5) = \frac{5}{2}$ not integer

Property 6 is not satisfied ∴ the set is not a vector space

Some important vector spaces
- $\mathbb{R}$ = the set of all real numbers
- $\mathbb{R}^2$ = the set of all ordered pairs
- $\mathbb{R}^3$ = the set of all ordered triples
- $\mathbb{R}^n$ = the set of all ordered n-tuples
- $M_{m \times n}$ = the set of all $m \times n$ matrices
- $M_{n \times n}$ = the set of all square matrices
- P_n = the set of all polynomials of degree $\leq n$
- $C(-\infty, \infty)$ = the set of all continuous functions defined on the open interval $(-\infty, \infty)$

Subspaces of Vector space

$S = \{1, 2, 3, 4, 5, 6\}$ Large set
$H = \{x : 1 \leq x \leq 6, x \text{ is an even integer}\}$
$H = \{2, 4, 6\} \equiv$ Subset of S $\quad H \subset S$
$W = \{x : 1 \leq x \leq 6, x \text{ is an odd integer}\}$
$W = \{1, 3, 5\} \equiv$ Subset of S $\quad W \subset S$

A subspace of a vectors space (V) is a subset (H) of (V) that has 3 properties:-
a) The zero vector of (V) is in (H)
b) (H) is closed under vector addition $(u, v \in H,\ u + v = \in H)$
c) (H) is closed under scalar multiplication $(u \in H,\ cu \in H \quad c \text{ is scalar})$